Pre-Lab Experiment ???
Fourier analysis, beats and Doppler shift

1 Introduction

“Beats” refers to the situation where two periodic signals, having nearly the same frequencies, interfere. Since the frequencies are not equal, the signals are, at times, in phase and, at other times, out of phase. The resulting superposed signal has amplitude which alternates between large and small, depending on the relative phases of the two signals.

Most of us have experienced the Doppler effect, probably in a situation where a car or motorcycle passes us at a relatively high speed. The sound, specifically the pitch or frequency, of the engine changes as the vehicle passes us. The vehicle begins by approaching us, but after it has passed, it is receding from us. This characteristic rise in frequency as the vehicle approaches and fall in frequency as the vehicle recedes is called the Doppler effect. The Doppler effect occurs any time either the source, the observer or both are in motion. In this experiment, we’re going to study the Doppler effect using a moving source; the moving observer version is similar.

2 Beats

Consider two sinusoidal signals:

\[ S_1(t) = A \cos(2\pi f_1 t) \]
\[ S_2(t) = A \cos(2\pi f_2 t) \]

which are added together. The result is

\[ S(t) = 2A \cos(2\pi \bar{f} t) \cos(2\pi \frac{f_{\text{beat}}}{2} t) \]

where \( \bar{f} = \frac{f_1 + f_2}{2} \) and \( f_{\text{beat}} = |f_1 - f_2| \). This is illustrated in the figure below. Verify this result by algebraically adding the two cosine functions above.
3 Doppler effect

3.1 Theory

Sound waves leaving a point source (S) expand in spherical waves centered on the source. Typically, such waves are represented by concentric circles, where the circles correspond to the wavefronts (high pressure regions), as indicated in the figure:
The observer detects sound waves of wavelength $\lambda$ (the distance between circles) and the same frequency as emitted by the source:

\[
\lambda = vT
\]
\[
\lambda = v/f
\]
\[
\lambda f = v.
\]

where $T$ is the period of the sound wave, and $v$ is the speed of sound.

If the source ($S$) is moving and the observer is stationary, there is a change in the *apparent* wavelength of the sound waves, and its value depends on where the observer is positioned:

If the source is moving directly towards the observer as pictured, determine the distance (call it $\lambda'$) between adjacent wavefronts, in terms of $v$, $v_s$, and $T$:

The time between adjacent wavefronts is still $T = 1/f$. Determine $\lambda'$ in terms of $v$, $v_s$, and $f$: 
Using the equations above, determine the new frequency, \( f' \), in terms of the speed of sound \( v \) and the apparent wavelength \( \lambda' \):

Now, replace \( \lambda' \) with the expression you found above in terms of \( v, v_s \), and \( f \). You should see that \( f' \) is directly proportional to \( f \):

Repeat the same steps for the source receding from the observer:

Do these two expressions make sense to you? You should find that the only difference between the source approaching and the source receding is that \( v_s \to -v_s \).
The final result, where the source and/or observer are moving is:

\[ f' = f \frac{(v \pm v_o)}{(v \mp v_s)}. \]  

(4)

The signs above are independent of each other! Use an upper sign for the source or observer approaching the other and a lower sign for the source or observer receding from the other. As an example, consider the source being a police car, heading east at 100 km/h and the observer is in another car also heading east at 100 km/h, trying to escape the from the police. In this example, the source is approaching the observer (upper sign) while the observer is receding from the source (lower sign) and the detected frequency is

\[ f' = f \frac{(v - v_o)}{(v - v_s)} \]

\[ f' = f. \]  

(5)

3.2 Qualitative Understanding

The Doppler shift equations derived above work for any wave moving in a medium, when the motions of the source and detector are along the line joining the source and detector. Most waves do require a medium to propagate, for example sound waves need air, or some liquid or solid, to travel from place to place. In the equations above, \( v, v_o \) and \( v_s \) are speeds relative to the medium; if the air is moving, \( v_o \) and \( v_s \) should be the air speeds rather than the ground speeds.

Light waves, or more generally Electromagnetic (EM) waves do not require a medium to propagate. Because of that, the Doppler shift for light is quantitatively different from that for sound (that it, the equations are different), but qualitatively they are the same.

Before coming to lab and beginning the experimental task, take a few minutes to convince yourself that you understand the Doppler shift qualitatively. Think back on your personal experiences; does the equation make sense in a general way? What situations will lead to an increase in frequency, or pitch? What situations will lead to a decrease in frequency, or pitch? Be ready to answer some simple qualitative questions from your instructor.

3.3 Quantitative Understanding

Again, before beginning the experimental task, take a few minutes to convince yourself that you can use the Doppler shift equation correctly. Imagine that you have 2 cars, moving at either 0m/s or 30m/s (close to 70 mph), one of which has a 1000 Hz siren, and for this simple exercise, choose the speed of sound to be 300m/s to simplify the arithmetic. There are eight different scenarios possible: two with stationary source but moving observer; two with a stationary observer but moving source; one with both cars moving away from each other; one with both cars moving towards each other; two with both cars moving in the same direction. Are any of these surprising to you? Which combination gives the largest shift? Which gives the smallest shift?

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