Experiment 8
Rotational Dynamics

Score: ___________/100

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Object

To study angular motion and the concept of moment of inertia; in particular, to determine the effect of a constant torque upon a disk free to rotate with, to measure the resulting angular rotation and to determine the moment of inertia of the disk.

Apparatus

Moment of inertia apparatus, clamps and supports, weights, thread, hanger, vernier, meter stick, scale, smart pulley, Science Workshop and Excel.

Theory

Newton’s second law in rotational form is given by:

\[ \tau = I\alpha \quad (1) \]

et torque (\(\tau\)) is equal to moment of inertia (\(I\)) times angular acceleration (\(\alpha\)).

In the following discussion, the distance from the center of rotation will be denoted \(R_c\). The relationship between the linear acceleration of the edge of a disk (and consequently the linear acceleration of a falling mass attached to the disk by a string) is

\[ a = R_c\alpha. \quad (2) \]

There is some friction between the disk and its support; it is actually a frictional torque. It will be approximately constant. We’ll measure it in this experiment. Consider first, the falling mass; tension in the string is pulling up, the weight is pulling down. Draw a fbd for the weight, and show that (5 points):

\[ T = mg - ma = mg - m\alpha R_c. \quad (3) \]
The torque due to the tension ($\tau_T$) is:

$$\tau_T = TR_c = (mg - m\alpha R_c)R_c$$  \hspace{1cm} (4)

since the tension acts a distance $R_c$ from the center of rotation. Finally, the net torque is simply the difference between $\tau_T$ and $\tau_f$, and it leads to the acceleration of the disk.

Draw a fbd for the flywheel, and show that the sum of torques is (10 points):

$$\tau_{net} = TR_c - \tau_f = mgR_c - mR_c^2\alpha - \tau_f$$  \hspace{1cm} (5)

and that the angular acceleration is given by (5 points):

$$\alpha = \frac{mgR_c}{I + mR_c^2} - \frac{\tau_f}{I + mR_c^2}$$  \hspace{1cm} (6)
Activity 1 (15 points total)

1) Measure the mass of the disk $M$ (it can be removed easily from the stand), and the dimensions outlined in the picture below:

![Diagram of a disk with dimensions](image)

Record all values here (5 points):

2) You can think of the disk as being composed of 3 parts: the 2 “wings” and 1 central disk. The total moment of inertia is then given by:

$$ I = \frac{1}{2} M_1 R_1^2 + \frac{1}{2} M_2 R_2^2 + \frac{1}{2} M_3 R_3^2 \tag{7} $$

where the $R_i$ are just $d_i/2$. The mass of the different parts is related to the total mass of the whole disk and it’s fraction of the total volume. The volume of a cylinder is given by $V = \pi LR^2$, so the mass of a given part is:

$$ M_i = M \frac{\pi L_i d_i^2/4}{\pi L_1 d_1^2/4 + \pi L_2 d_2^2/4 + \pi L_3 d_3^2/4} = M \frac{L_i d_i^2}{L_1 d_1^2 + L_2 d_2^2 + L_3 d_3^2}. \tag{8} $$

Calculate the moment of inertia of the disk (be careful of units! everything should be in meters and
kilograms! (10 points):

Activity 2 (25 points total)

1) Wrap the string around the central part of the disk, run the string through the smart pulley, and adjust the height of the smart pulley holder such that the string between the disk and the pulley is horizontal.

2) Define calculations of \( \theta = (x \text{ position})/R_c \) and \( \omega = (\text{velocity})/R_c \).

3) For masses of 50, 60, 70, 80 and 90 grams, collect data on the motion of the disk. Since, for a given run, \( \alpha \) should be constant \( \theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2 \) should work, and you can extract \( \alpha \) from a graph of \( \theta \) vs. \( t \).
4) Verify that \( mR_c^2 << I \) for each of the masses used, so we can approximate \( \alpha = \frac{mgR_c}{I} \). \( \tau \) (5 points):

5) Import your \( \alpha \) and \( m \) values into Excel, graph \( \alpha \) on the y axis and \( m \) on the x axis, and determine the slope \( (= \frac{gR_c}{I}) \) and intercept \( (= -\frac{\tau}{I}) \). Print and include the graph from Excel (10 points).

6) Using known values for \( g \) and \( R_c \), does the slope of this line give the same moment of inertia as your earlier calculation? What is the value of \( \tau \)? (10 points)

Activity 3 (25 points total)

1) Wrap the string around one of the wings of the disk, run the string through the smart pulley, and adjust the height of the smart pulley holder such that the string between the disk and the pulley is horizontal.

2) Define calculations of \( \theta = \frac{\text{(x position)}}{R_c} \) and \( \omega = \frac{\text{(velocity)}}{R_c} \). Keep in mind that you have changed \( R_c \).

3) For masses of 50, 100, 150, 200 and 250 grams, collect data on the motion of the disk. Since, for a given run, \( \alpha \) should be constant \( \theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2 \) should work, and you can extract \( \alpha \) from a graph of \( \theta \) vs. \( t \).
4) Verify that \( mR_c^2 \ll I \) for each of the masses used, so we can approximate \( \alpha = \frac{mgR_c}{I} - \frac{\tau_f}{I} \) (5 points):

5) Import your \( \alpha \) and \( m \) values into Excel, graph \( \alpha \) on the y axis and \( m \) on the x axis, and determine the slope (\( = \frac{gR_c}{I} \)) and intercept (\( = -\frac{\tau_f}{I} \)). Print and include the graph from Excel (10 points).

6) Using known values for \( g \) and \( R_c \), does the slope of this line give the same moment of inertia as your earlier calculation? What is the value of \( \tau_f \)? (10 points)

**Final Analysis (15 points)**

- How do the values of \( \tau_f \) from Activities 2 and 3 compare? How should they be related?
- How do the values of \( I \) from Activities 2 and 3 compare? How should they be related?
- How do the values of \( I \) from Activities 2 and 3 compare with the value of \( I \) from Activity 1? Can you think of any construction techniques that would make the method of Activity 1 invalid?

**Acknowledgements**

Project development was supported by an NSF-ILI Grant Interactive, Microcomputer-Based Undergraduate Physics Laboratory Grant No. DUE-9851024. Any opinions, findings, conclusions or recommendations expressed in this document are those of the author, and do not necessarily reflect the views of the National Science Foundation.