THE NOT SO SIMPLE PENDULUM

INTRODUCTION:

This laboratory experiment is used to study a wide range of topics in mechanics like velocity, acceleration, forces and their components, the gravitational force, the tension force, the centripetal force, the gravitational potential energy, the kinetic energy, the conservation of mechanical energy, the energy loss to friction and of course the concept of period of pendulum. It requires a force probe at the point of support, which is made free to rotate with the swinging of the pendulum and a position sensor to look at the motion of the pendulum bob. This experiment may be used as a final experiment to wrap up the study of mechanics.

EXPERIMENTAL SETUP:

The critical point in this setup is to have a rigid support and frictionless pivot at the point of support. A horizontal rod (1/2 " diameter) mounted with a clamp on a platform attached to the wall provided the rigid support. The force probe has a mounting hole in its body at one end and it is hung on the horizontal rod through this hole. The friction between the rod and the force probe was a big problem and made the motion jerky. The friction was reduced significantly by wrapping the rod with Teflon paper at the position of support. The motion became smooth with the use of Teflon paper. A heavy spherical mass (approximately 1.6 kilogram) was tied to one end of a string and the other end was attached to the hook of the force probe. The heavy mass pendulum bob insured that the force probe would swing in line with the string and the system could be viewed as simple pendulum because the rotational inertia contribution due to the force probe was small compared to the rotational inertia of the bob. The spherical shape of the bob also reduced the air drag. The motion sensor was set at approximately 60 cm from the mean position of the pendulum and at the same level as the average vertical position of the mass when the mass is swinging. The length of the pendulum from the point of support to the center of the hanging mass was 1.02 m. The force probe was calibrated with standard 1 kg mass. The pendulum was started by displacing the mass by 10 cm horizontally and releasing it. The position sensor recorded the horizontal position of the mass and the force probe recorded the tension force along the direction of the
string in real time. The Science Workshop software was used to record these variables and analyze the data. The analysis is discussed in detail below.

**Force Probe**

![Force Probe](image1.png)

**Position Sensor**

![Position Sensor](image2.png)

Fig. 1. The force probe showing the mounting hole on the right end and the hook on the left.  

Fig. 2. The position sensor measure the position of the pendulum bob.

**The Pendulum**

![Pendulum Diagram](image3.png)

Fig. 3. The layout of the experiment shows the simple pendulum string and ball mass hooked to the force probe hanging on the rigid support rod. The position sensor is on the right.
ANALYSIS:

Position, Velocity and Acceleration:

The deriving force for the motion of the mass is mg\text{Sin}(\theta). For small angles this can be written as mgx/L which shows that the force is proportional to the horizontal position x. The acceleration, gx/L is also proportional to x. The graph of acceleration versus position in Fig. 4 shows this behaviour very clearly.

Fig. 4 The third graph in this figure shows the linear relationship between the acceleration and the position as it should for restoring force is proportional to the displacement.

The plot of position, velocity and acceleration versus time on the same graph in Fig. 5 show how they are related to each other. Specially the first derivative relationship between position and velocity and between velocity and acceleration. Where the position is at a maximum or minimum the velocity is zero and where the velocity is at a maximum or minimum the acceleration is zero. Conversely the integral of acceleration from the starting point is velocity and at the end of each complete cycle in acceleration plot the net area is zero and the velocity is zero. Similar behavior is observed for velocity and position graph.
Gravitational Force, Tension Force and Centripetal Acceleration:

The forces acting on the mass at any instant in its motion is shown in the free body diagram in Fig. 6 below.

![Free Body Diagram](image)

Fig. 6 The free body diagram of the pendulum bob shows the gravitational force $mg$, the tension force $T$ and the centripetal acceleration $a_c$. The components of the gravitational force are also shown.

Applying Newton's second law along the direction of tension force on gets
\[ T - mg \cdot \cos(\theta) = ma_c = \frac{(mv^2)}{r} \]

\[ T = mg \cdot \cos(\theta) + \frac{(mv^2)}{r} \]  \hspace{1cm} (1)

where \( v \) is the velocity of the bob and \( a_c \) is the centripetal acceleration. Also from Fig. 3 on can see that:

\[ \sin(\theta) = \frac{x}{L} \]  \hspace{1cm} (2)

\[ \cos(\theta) = \sqrt{1 - \sin^2(\theta)} = \sqrt{1 - \left(\frac{x}{L}\right)^2} \]  \hspace{1cm} (3)

\[ v_x = v \cdot \cos(\theta) \]  \hspace{1cm} (4)

Since the position detector measures \( x \) and \( v_x \) one can get \( v \) from equation 4. Substitution from equations 3 and 4 in 1 will give the predicted value of the tension \( T \) in the string. This is to be compared with the tension measured by the force probe. The Science Workshop data acquisition and display software has facility to calculate the tension \( T \) from recorded variables. The measured tension by the force probe and the calculated values are plotted in real time in Fig. 7.

**Fig. 7** The top graph is the force measured by the force probe. The middle graph is the calculated value of the tension in the string and the bottom graph is the difference of graph one and two.
The difference between the calculated value and the value measured by the force probe is shown in the third graph. This is close to zero as expected.

**Kinetic Energy, Gravitational Potential Energy, Conservation of Mechanical Energy and Energy loss due to Friction:**

Taking the gravitational potential energy to be zero at the lowest point of the pendulum bob one can write the gravitational potential energy as:

\[
PE = mgh = mg(L - L\cos(\theta)) = mgL(1 - \sqrt{1-(x/L)^2}) \quad (5)
\]

and the kinetic energy as:

\[
KE = (1/2)mv^2 = (1/2)m(v_x/\cos(\theta))^2 \quad (6)
\]

The plot of KE and PE in real time is shown in the first and the second graph in Fig. 8 respectively. Where the KE is maximum, the PE is minimum and where the PE is maximum the KE is minimum as expected. The third graph in this figure shows the sum of KE and PE, which is equal to the total mechanical energy.

![Graphs showing KE, PE, and total mechanical energy](image)

**Fig. 8** The first graph is the Gravitational Potential Energy and the second graph is the kinetic energy. The third graph is the total mechanical energy. The total energy line goes down slowly because of the loss to friction.
This total mechanical energy is constant over small time intervals but decreases gradually with time. This decrease is due to the loss of energy due to friction. The slope of this graph is a measure of power loss due to friction.

**Period of the Simple Pendulum:**

Finally one can calculate the period of the simple pendulum from the following equation:

\[
T = 2\pi\sqrt{\frac{L}{g}} \quad (7)
\]

It is found that the measured value of the period agrees more closely with the predicted value as the amplitude of the oscillation decreases.